ME 314 - Engineering Design : Mechanical Components
Lecture 3

Note Title

3.9 Beam Loading (Shear Force & Bending Moment)

A beam is an element that carries loads transverse to its long axis and may carry loads in the axial direction as well.

Types of Support

Simply Supported Beam:

Cantilever Beam:

Overhung Beam:

Indeterminate Beam:
(has more supports than needed)

Types of Load

Sign Convention

Relation between q(x), V(x), and M(x)

Shear force, V, the bending moment, M, and the loading function, q, are related by

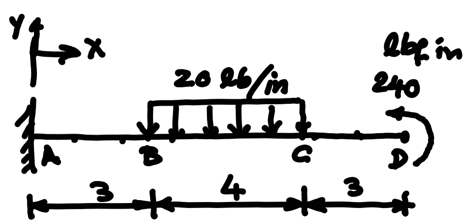
Given q(x), integrate between, say, x_A and x_B:

Change in shear force from A to B equals the area of the loading diagram

Change in moment from A to B equals the area of the shear force diagram between x_A and x_B.

Singularity Functions

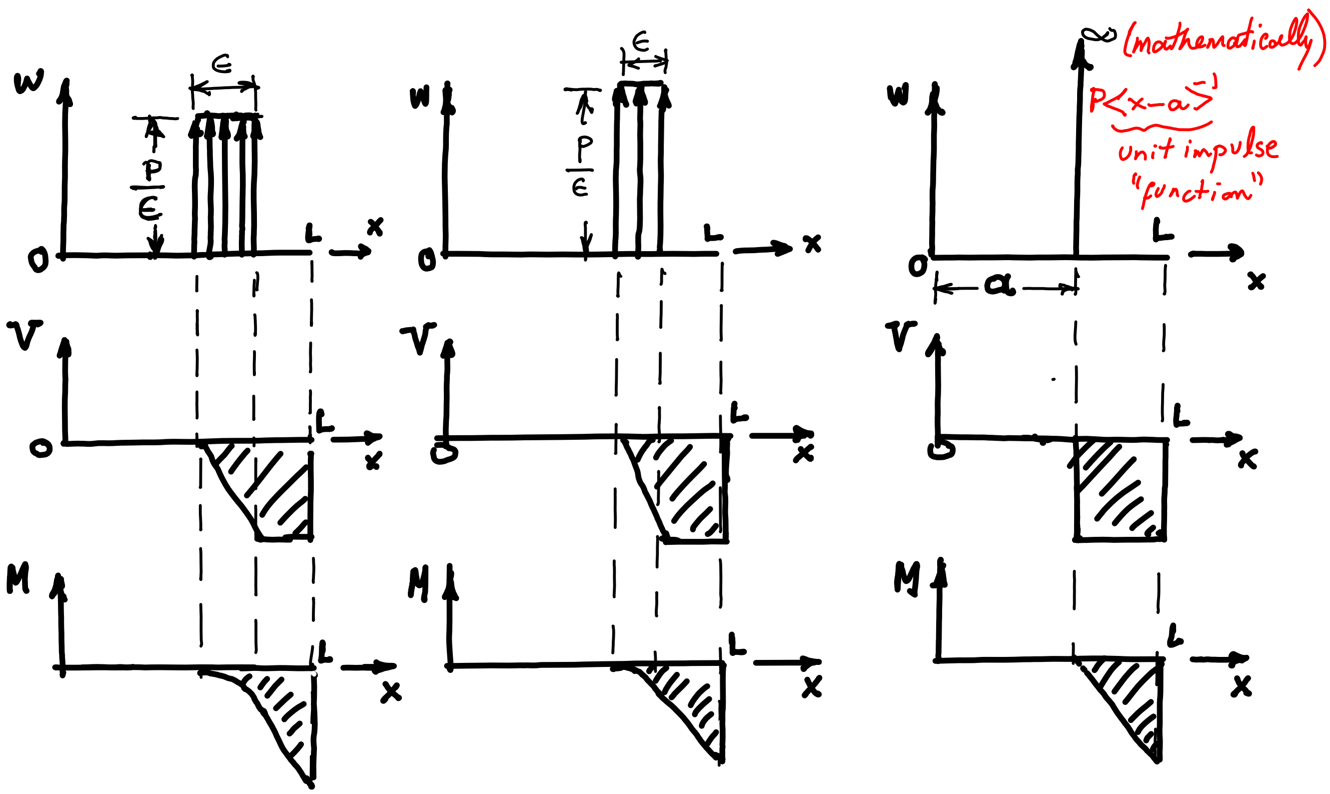
These are special class of functions that are useful for dealing with discrete functions such as point loads or segments of distributed loads.



For a problem such as the one shown in the figure, the usual approach of making cuts and finding the shear and moment expressions would necessitate that shear and monent expressions be constructed for each of the three intervals $0 \leq x \leq 3$, and $3 \leq x \leq 7$. Singularity functions permit us to write a single expression for the moment that is valid over the entire length of the beam. This is particularly helpful in calculating the bending deflection of a beam by integrating the expression for moment twice according to the beam equation EI y'' (x) = M. This procedure will be reviewed later in Section 4.10.

Concentrated Loads

Consider a distributed load *w* that acts over a relatively small segment Є of a beam as shown.

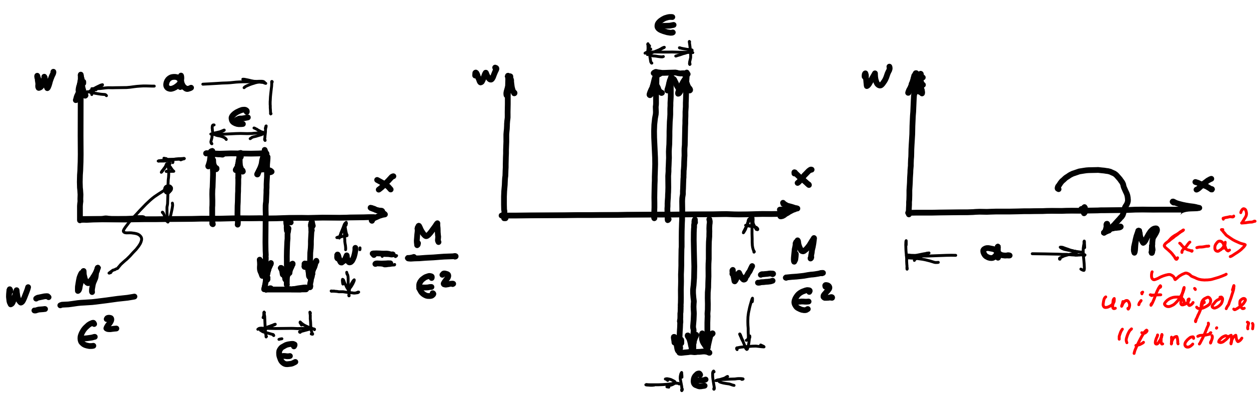


The singularity function is not actually a "function," rather only an indication that something rather unusual is being modeled, namely, a finite load P applied over an extremely small distance Є.

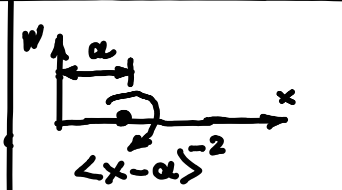
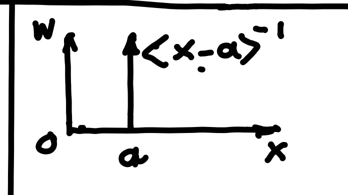
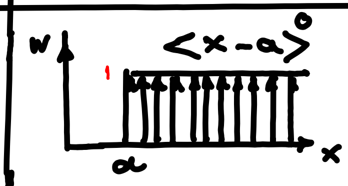
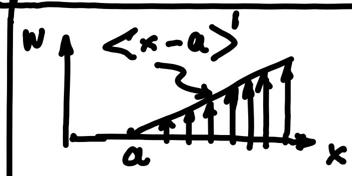
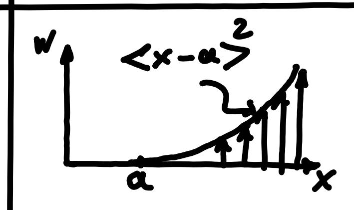
Other symbols used for $<x-a>^{-1}$ are $\delta(x,a)$ and $\delta(x-a)$. These symbols are often referred to as the Dirac delta function or the unit impulse function.

Concentrated Moments

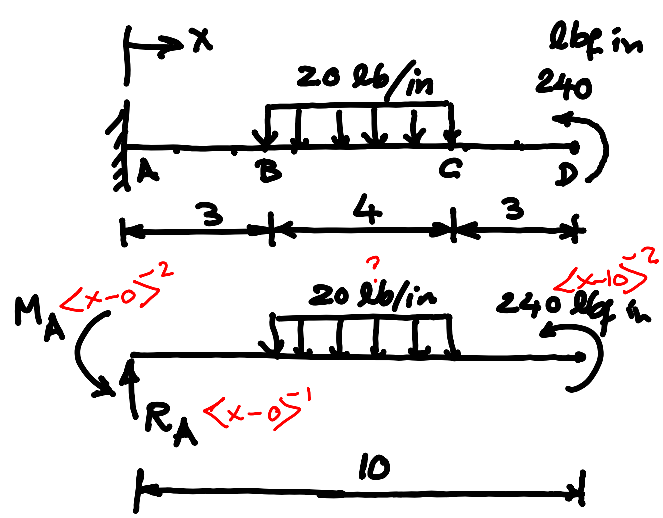
Now consider the loading shown in figure, where two adjacent segments of the beam are subjected to loads $w\epsilon$ that have opposite sense.



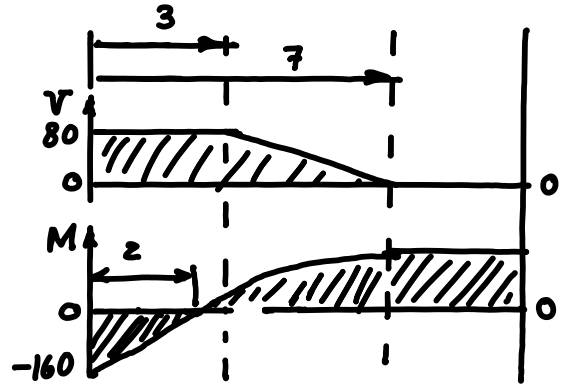
Singularity Functions

Name	Definition	Graph	Integral
Unit dipole function at point *a*	$\langle x-a\rangle^{-2} = \begin{cases} 0 & \text{if } x \neq a \\ +\infty & \text{if } x = a \end{cases}$		$\int \langle x-a\rangle^{-2}dx = \langle x-a\rangle^{-1}$
Unit impulse function at point *a*	$\langle x-a\rangle^{-1} = \begin{cases} 0 & \text{if } x \neq a \\ +\infty & \text{if } x = a \end{cases}$		$\int \langle x-a\rangle^{-1}dx = \langle x-a\rangle^{0}$
Unit step function at point *a*	$\langle x-a\rangle^{0} = \begin{cases} 0 & \text{if } x < a \\ 1 & \text{if } x \geq a \end{cases}$		$\int \langle x-a\rangle^{0}dx = \langle x-a\rangle^{1}$
Ramp function at point *a*	$\langle x-a\rangle^{1} = \begin{cases} 0 & \text{if } x < a \\ x-a & \text{if } x \geq a \end{cases}$		$\int \langle x-a\rangle^{1}dx = \frac{1}{2}\langle x-a\rangle^{2}$
Parabolic function at point *a*	$\langle x-a\rangle^{2} = \begin{cases} 0 & \text{if } x < a \\ (x-a)^2 & \text{if } x \geq a \end{cases}$		$\int \langle x-a\rangle^{2}dx = \frac{1}{3}\langle x-a\rangle^{3}$

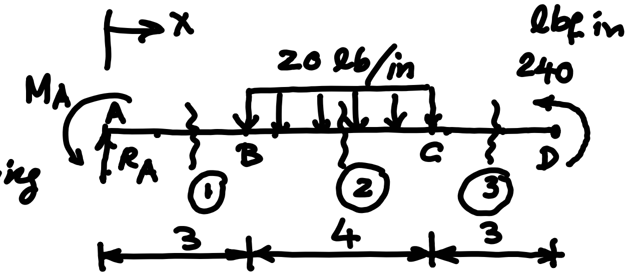
Example: A cantilever beam is loaded as shown. Derive the shear-force and bending-moment relations, and the support reactions.



Caution: **Note that both M_A and the 240 lb-in moments are counterclockwise and negative singularity functions; however, by the convention shown on page 4 above, M_A and the 240 lb-in moments are negative and positive moments, respectively, when we draw the moment diagram.**


3
7
V
80
0
0
M
2
0
0
-160

Method of Cuts:



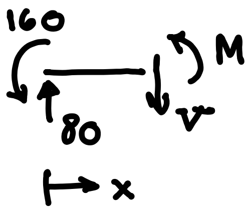
We review this method by solving the above example.

Reactions : $\circlearrowleft + \sum M_A = 0$ $\quad M_A - (20)(4)(5) + 240 = 0$

$M_A = 160$ lbf.in

$+\uparrow \sum F_y = 0$ $\quad R_A - (20)(4) = 0$, $R_A = 80$ lbf.in $\uparrow$

cut ①

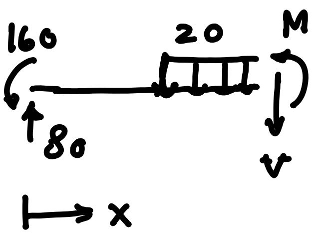


$\circlearrowleft + \sum M_{cut} = 0$: $+M - 80x + 160 = 0$

$+\uparrow \sum F = 0$: $-V + 80 = 0$

$\underline{V = 80}$, $\underline{M = -160 + 80x}$; $\underline{0 \leq x \leq 3}$

cut ②

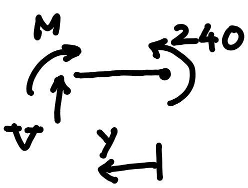


$\circlearrowleft + \sum M_{cut} = 0$: $+M + 160 - 80x + 20\frac{(x-3)^2}{2} = 0$

$+\uparrow \sum F = 0$: $-V + 80 - 20(x-3) = 0$

$\underline{V = 140 - 2x}$, $\underline{M = -160 + 80x - 10(x-3)^2}$, $\underline{3 \leq x \leq 7}$

cut ③



, $\circlearrowleft + \sum M_{cut} = 0$: $-M + 240 = 0$

$+\uparrow \sum F = 0$: $+V = 0$

$\underline{V = 0}$, $\underline{M = 240}$, $\underline{7 \leq x \leq 10}$

3.8 Impact Loading

Please study this section on your own.